Due Date: September 2, 2005

1. Write matlab code to implement softmax regression for arbitrary dimension.

2. Show that the moment generating function of an exponential-family distribution in terms of $T(x)$ is as follows:

$$E(e^{s^T T(x)}) = e^{A(s+\eta)-A(\eta)} \tag{1}$$

3. The Dirichlet distribution is a continuous distribution on the K-simplex $\theta = (\theta_1, \theta_2, \ldots, \theta_K$ with $\theta_i \geq 1$ for $i = 1, \ldots, K$ and $\sum_{i=1}^K \theta_K = 1$.

$$p(\theta|\alpha) = \frac{\Gamma(\sum_{i=1}^K \alpha_i)}{\Pi_{i=1}^K \Gamma(\alpha_i)} \theta_1^{\alpha_1 - 1} \theta_2^{\alpha_2 - 1} \ldots \theta_K^{\alpha_K - 1} \tag{2}$$

   where $\alpha_i \geq 0$ are parameters.

   (a) Compute $E[\theta_k]$. [Hint: do it directly.]

   (b) Compute $Cov[\theta_j \theta_k]$. [Hint: do it directly.]

   (c) Compute $E[\log \theta_k]$.

4. Let $\theta \sim \mathrm{Dir}(\alpha)$. Consider multinomial random variables $(X_1, X_2, \ldots, X_N)$, where $X_n \sim \mathrm{Mult}(\theta)$ for each n, and where the X_n are assumed conditionally independent given θ. Now consider a random variable $X_{\text{new}} \sim \mathrm{Mult}(\theta)$ that is assumed conditionally independent of $(X_1, X_2, \ldots, X_n)$ given θ. Compute:

$$p(x_{\text{new}}|x_1, x_2, \ldots, x_N, \alpha) \tag{3}$$

   by integrating over θ. [Hint: your result should take the form of a ratio of gamma functions.]